Polarization of Nuclear Spins from the Conductance of Quantum Wire

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We devise an approach to measure the polarization of nuclear spins via conductance measurements. Specifically, we study the combined effect of external magnetic field, nuclear spin polarization, and Rashba spin-orbit interaction on the conductance of a quantum wire. Nonequilibrium nuclear spin polarization affects the electron energy spectrum making it time-dependent. Changes in the extremal points of the spectrum result in time-dependence of the conductance. The conductance oscillation pattern can be used to obtain information about the amplitude of the nuclear spin polarization and extract the characteristic time scales of the nuclear spin subsystem.

The promise of spintronics and quantum computing has motivated recent theoretical and experimental investigations of spin-related effects in semiconductor heterostructures [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Nuclear and electron spins have been considered as candidates for qubit implementations in solid state systems [1, 2, 3, 4, 5, 6]. The final stage of a quantum computation process involves readout of quantum information. In the case of a spin qubit one would have to measure the state of a single spin. Yet, in spite of recent efforts in this field, a single nuclear spin measurement is still a great challenge.

There are several proposals for single- and few-spin measurement. For example, a change of the oscillation frequency of a micro-mechanical resonator (cantilever) [11] is used. Another possibility to obtain information about a qubit state lies in the measurement [12] of current or its noise spectrum in a mesoscopic system (e.g., quantum wire, quantum dot, or single electron transistor) coupled to a qubit [13, 14, 15]. Significant progress in spin measurements has been made using magnetic resonance force microscopy [16], which presently allows one to probe the state of 100 fully polarized electron spins. Recently, an experimental architecture to manipulate the magnetization of nuclear spin domains was proposed [5, 6].

The present work demonstrates that a relatively small ensemble of nuclear spins can significantly influence transport through a quantum wire (QW). This offers a new detector design, with the operation based on a new effect arising as a consequence of the combined influence of the spin-orbit interaction and nuclear spin polarization on the electron subsystem. Recent progress in investigations of QWs [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28] makes them a promising nanoscale device component.

We consider transport through a QW in the presence of an external in-plane magnetic field, Rashba spin-orbit coupling [29] and a nonequilibrium nuclear spin polarization. We assume that the external magnetic field is directed along a wire. If the nuclear spin polarization has a non-zero component perpendicular to the external field at the initial moment of time (i.e. the two vectors are not aligned), then we will demonstrate that the conductance of the wire exhibits damped oscillations. These oscilla-

tions are a direct consequence of the interplay between the evolving field of the nuclei and spin-orbit interaction experienced by the conduction electrons in the QW. Our results reveal that the damping times of these oscillations are of the order of the longitudinal and transverse relaxation times of the nuclear spins, while the frequency of the oscillations is directly related to the nuclear spin precession. With presently available QWs, the number of nuclear spins in the region of the QW can be quite large. However, experimental realizations of our proposed system will yield valuable insights into the physics of spin dynamics and measurement and will advance the future implementations of few-spin electronic devices.

The system under investigation is depicted in Fig. 1. The two-dimensional electron gas is split into two parts by a potential applied to the gate electrodes. The narrow constriction between the gates then forms a "dynamic" quantum wire. Let us define a coordinate system such that the direction of the electron transport through the wire is in the x-direction and lateral confinement is in the y-direction. We assume that an external magnetic field is applied in the x-direction. We will also consider an ensemble of nuclear spins polarized locally in the region of the wire. Experimentally, this can be accomplished by means of optical pumping or other spatially-selective techniques [30, 31, 32].

Once the nuclear spins are polarized, the charge carrier spins feel an effective hyperfine field, B_n, which lifts the
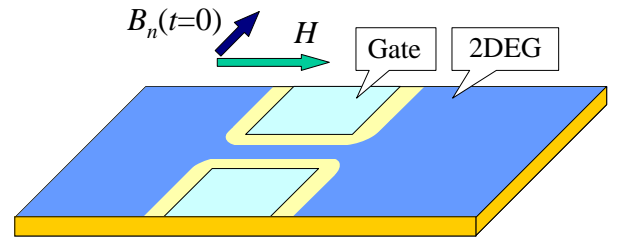

FIG. 1: Quantum wire with an applied magnetic field in the x direction, and an effective nuclear hyperfine field initially pointing in the y direction.
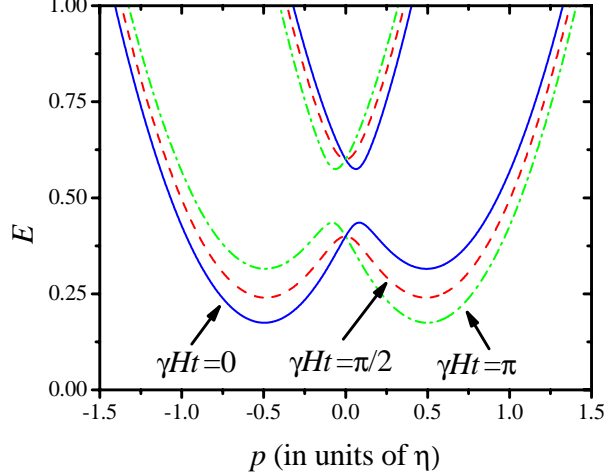
FIG. 2: The lowest-energy sub-bands, in units of $\hbar\omega$, as functions of $p = \hbar k$, for three different times. Here $\eta \equiv \sqrt{2m^*\hbar\omega}$. The two sets of curves correspond to spin $\pm$.

spin degeneracy. The maximum nuclear field in GaAs can be as high as $B_n = 5.3$T in the limit that all the nuclear spins are fully polarized [33]. This high level of nuclear spin polarization has been achieved experimentally [30, 31]. Typically, natural semiconductor materials contain at least a small fraction of one elemental isotope with non-zero nuclear spin I [34, 35]; for example, ^{69}Ga (natural abundance 60.1%), ^{71}Ga (39.9%), ^{75}As (100%), all have nuclear spin $I = 3/2$.

We will consider the effect that the precession and decay of the nuclear spin polarization have on the current through the QW. In order to observe this effect, the nuclear spin polarization should not be collinear with the applied magnetic field. We will assume that at the initial moment of time all the nuclear spins are polarized along x, and then only *one kind* of nuclear spins (those of the same isotope) are selectively rotated to point in the y-direction, e.g., by a radio-frequency NMR pulse [36].

The evolution of the nuclear magnetization can be described phenomenologically by the Bloch equations [36]. Since the effective magnetic field experienced by the conduction electron spins due to the nuclear spin polarization is proportional to the nuclear magnetization, we can write the Bloch equations as

$$\frac{d\vec{B}_n^i}{dt} = \gamma_i \vec{B}_n^i \times \vec{H} - \frac{B_{n,y}^i \hat{y} + B_{n,z}^i \hat{z}}{T_2^i} - \frac{B_{n,x}^i - B_0^i}{T_1^i}\hat{x}, \quad (1)$$

where the index $i = 1, \ldots, p$ denotes different types of nuclear spins, γ_i denotes the gyromagnetic ratios, B_0^i gives the equilibrium values for the effective magnetic fields of the nuclear spins, $T_{1,2}^i$ are the longitudinal and transverse spin relaxation times, respectively. The total magnetic field due to the polarized nuclear spins is defined as $\vec{B}_n = \sum_{i=1}^p \vec{B}_n^i$. The equilibrium (thermal) value of the effective magnetic fields B_0^i is rather small, and will be neglected in what follows. Assuming that only the nuclear

spin isotope with $i = 1$ was rotated in the y-direction at $t = 0$, we can easily solve the Bloch equations (1) to obtain the time dependence of the effective magnetic field of the spin-polarized nuclei,

$$B_{n,x}(t) = \sum_{i=2}^p B_n^i(t = 0)e^{-t/T_1^i} \quad (2)$$

$$B_{n,y}(t) = B_n^1(t = 0)e^{-t/T_2^1}\cos(\gamma_1 H t) \quad (3)$$

$$B_{n,z}(t) = -B_n^1(t = 0)e^{-t/T_2^1}\sin(\gamma_1 H t) \quad (4)$$

Here $B_n^1(t = 0)\hat{y}$ and $B_n^{2,\ldots,p}(t = 0)\hat{x}$ are the initial values of the effective magnetic fields. In what follows we will denote $\gamma \equiv \gamma_1$.

In the QW, the Hamiltonian for the conduction electrons can be written in the form,

$$H = \frac{p^2}{2m^*} + V(y) - i\alpha\sigma_y\frac{\partial}{\partial x} + \frac{g^*\mu_B}{2}\vec{\sigma}\cdot\vec{B}. \quad (5)$$

Here, $\vec{p}$ is the momentum of the electron, $V(y)$ is the lateral confinement potential due to the gates, μ_B and g^* are the Bohr magneton and effective g-factor, $\vec{\sigma}$ is the vector of the Pauli matrices, and $\vec{B} = \vec{B}_n + \vec{H}$. The effect of the external field $\vec{H}$ on the spatial motion is neglected, assuming strong confinement in the z direction. The third term in (5) represents the Rashba spin-orbit interaction for an electron moving in the x-direction [28]. We assume that the effects of the Dresselhaus spin-orbit interaction can be neglected [37].

All the time scales of the nuclear spin dynamics are much longer than the the electron traversal time through the QW, t_e. Therefore, we can assume that the electrons are subject to constant effective interactions as they pass through the QW. In solving the Schrödinger equation for the electrons, we can treat the time-dependence of the nuclear hyperfine fields quasi-statically. To justify this statement, let us compare the shortest nuclear time scale, the oscillatory period of the nuclear hyperfine field, t_n, with t_e. For example, for ^{69}Ga, the spin precession frequency is 10.7042 MHz in magnetic field of 1T [35], which corresponds to $t_n \simeq 0.1\mu$s. The traversal time can be estimated as $t_e \simeq L/v_f$, where L is the length of the QW and v_f is the electron Fermi velocity. For $L = 1\,\mu$m and $v_f \simeq 10^7$ cm/s, we get $t_e \simeq 10$ ps $\ll t_n$.

The eigenvalues of (5) can be written [28] as

$$E_{\ell,\pm}(k) = \frac{\hbar^2 k^2}{2m^*} + E_\ell^{tr} \pm \Gamma\sqrt{B^2(t) + \frac{2\alpha k B_y(t)}{\Gamma} + \left(\frac{\alpha k}{\Gamma}\right)^2}.$$
$$(6)$$

Here, $\pm$ refer to the spin direction, $\Gamma = g^*\mu_B/2$, and E_ℓ^{tr} is the ℓ th eigenvalue of $V(y)$. Assuming the parabolic confinement potential in the y direction, we have $E_\ell^{tr} = \hbar\omega(\ell + 1/2)$. The energy spectrum (6) at different moments of time is shown in Fig. 2, for the two spin-split sub-bands characterized by $\ell = 0$. The time-dependence introduced by the nuclear spin precession drives causes the energy bands to oscillate in time. The nonparabolic

structure of the sub-bands is due to the Rashba term in (5). The directional asymmetry of the energy spectrum shown in Fig. 2 is related to the fact that the nuclear spin polarization increases the effective Rashba field for electrons moving in one direction and decreases this field for electrons moving in the opposite direction. We note that the nuclear spin relaxation processes described by the exponential damping factors in (2-4) ultimately suppress the oscillatory behavior of the energy spectrum with time.

In order to calculate the conductance of the QW, we assume [22] that the applied voltage is small compared to $k_B T/e$, and that the transport through the QW is ballistic. Then the conductance, G, in the wire can be approximated by the linear response formula [22, 27]. For sub-bands with several local extremal points, to be labeled by ext, it was shown in [28] that

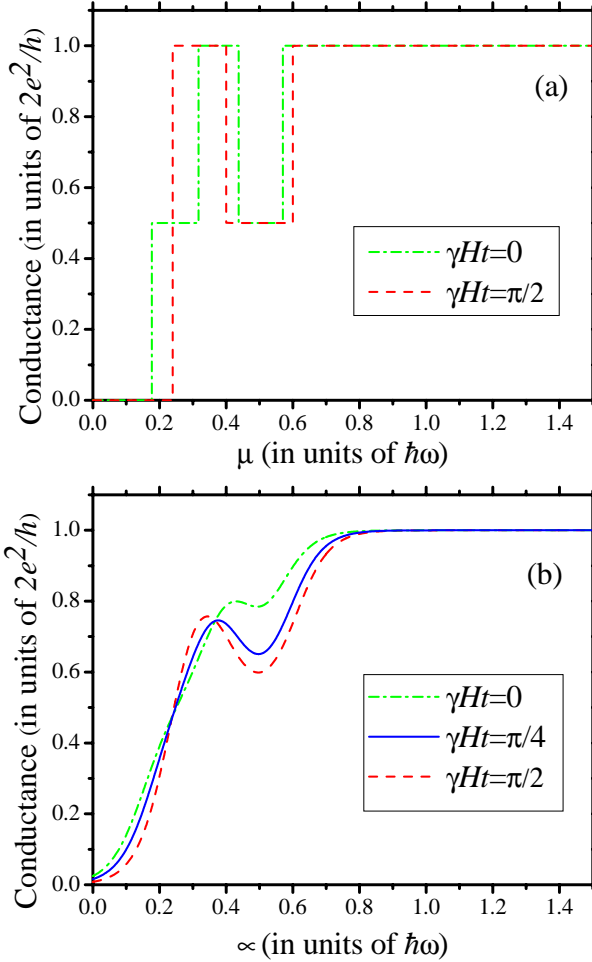$$G = \frac{e^2}{h} \sum_{\ell,\pm} \sum_{ext} \zeta^{ext} f(E_{\ell,\pm}^{ext}), \qquad (7)$$

FIG. 3: Time dependence of the conductance at (a) zero temperature and (b) finite temperature, as a function of the reservoir chemical potential, μ.
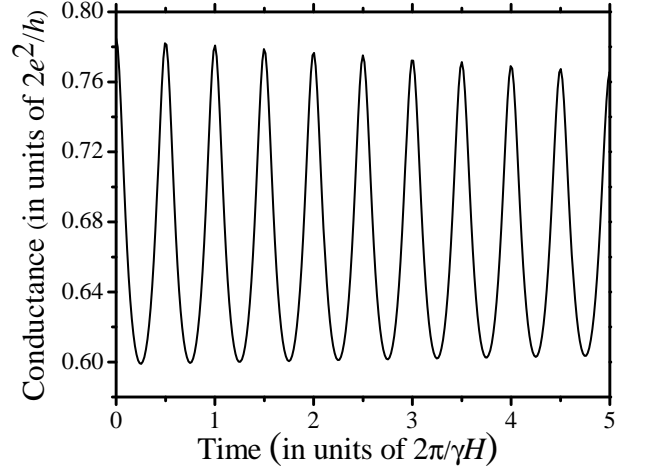

FIG. 4: Time dependence of the conduction at a finite temperature and at the fixed value of $\mu = 0.5\hbar\omega$.

where $f(E)$ is the Fermi-Dirac distribution function, ζ^{ext} is ± 1 for a minimum or maximum, respectively, and perfect transmission through the wire is assumed [22].

Figure 3 shows the numerically calculated conductance as a function of the reservoir chemical potential, measured from the bottom of the confining potential $V(y)$, at different times. The conductance curves in Fig. 3b are smooth due to the thermal broadening of the Fermi-Dirac distribution. As temperature is lowered, these plateaus will become sharper, as seen in Fig. 3a. It is interesting to note that for $\gamma H t = 0$ and π (not shown in Fig. 3) the plateaus are nearly identical. This is due to the fact that the conductance depends only on the *energies* of the extremal points and not their location. In the case of $\gamma H t = \pi/2$, the $2\alpha k B_y(t)/\Gamma$ term in (6) vanishes, thereby causing the energy spectrum to become symmetric with respect to the origin. Thus, according to (7), as the chemical potential of the reservoirs increases, the two minima will contribute $2e^2/h$ to the conductance. If the potential is increased further towards the local maximum point, the conductance will be lowered to e^2/h, which is illustrated in Fig. 3a.

In Fig. 4, the conductance at $\mu = 0.5\hbar\omega$ is shown. This serves to illustrate that with an appropriate choice of the parameters, one could observe large conductance oscillations in a QW. As noted before the oscillations are due to the precession of the nuclear hyperfine field and have the frequency $\omega_n = \gamma H$. However, these conductance oscillations are damped, and the envelope of this damping can be attributed mainly to the exponential decay of $B_y(t)$ on the time scale T_2.

In conclusion we have demonstrated that nuclear spin polarization can be monitored via the conductance measurements of a quantum wire. Precession and relaxation of polarized nuclear spins makes the energy spectrum of the quantum wire time-dependent with oscillating minima and maxima of the sub-bands. These oscillations could be observed in a conductance measurement at cer-

tain values of the gate voltage. We emphasize that this effect arises as a result of the interplay of the Rashba spin-orbit interaction, external magnetic field and nonequilibrium nuclear spin polarization.

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